

Polarization Dependence of the Bremsstrahlung Cross-Section in the Scattering of Electrons by Nuclei with Electromagnetic Multipole Moments

S. A. Al-Khateeb

*Department of Mathematics, Faculty of Science for Girls,
King Abdulaziz University, Jeddah, Saudi Arabi*

Abstract. The present work deals with one of the most important problem in the quantum electrodynamics, i.e the problem of bremsstrahlung process due to the interaction between lepton (electron) and the field of nuclei having electromagnetic multipole distributions. The detailed calculations for the cross-sections and the angular and energy distributions of the process are given in the first Born approximation of the quantum perturbation theory in two cases: With neglecting the polarizations of the photon and the electron, and with studying these polarizations. Applying the obtained formulae to some nuclei, the light nucleus (Be_4^9) and the heavier one (Al_{13}^{27}), to showing the effect of both circular photon polarization and longitudinal electron polarization on the bremsstrahlung cross-section, and comparing the results obtained for the two nuclei graphically.

I-Introduction

The Study of the bremsstrahlung process was firstly done by Bethe and Heitler^[1] in 1934, and many works about this process could have appeared in the period (1934-1954)^[2-4]. In these works, the polarizations of the photons of bremsstrahlung and of the electrons were not studied because of the complicity of the calculations in this case. The circular polarization of bremsstrahlung photons due to the collision of longitudinally polarized electrons with nuclei, was firstly investigated by Grodzins^[5] in 1959. At the same time, Olsen and Maximon^[6] studied the circular polarization of the bremsstrahlung photons experimentally. Motz

and Placious^[7] in 1960, studied the linear polarization of the bremsstrahlung photons, and in 1962, Bisi and others^[8] made some measurements about the dependence of the degree of circular polarization of bremsstrahlung on the electric and magnetic field and on the atomic number of the target nuclei. The intensity of bremsstrahlung due to the interaction of electric and magnetic interactions between electrons and target nucleus, had been calculated by Ginsberg and Pratt^[9] in 1964, and in the next year, Mork and Olsen^[10] investigated the polarization effect on bremsstrahlung, and showed that the study of polarization caused to increase the cross-section of the process. The study of bremsstrahlung due to the scattering of longitudinal polarized electrons by different nuclei was done by Neumcke^[11] in 1966, and after many years, Behncke and Nakel^[12] in 1974 calculated the linear polarization of photons in the scattering of electrons with carbon nucleus. In 1989, Caffo and others^[13], investigated the polarization of the scattered electrons and of the emitted photons in the bremsstrahlung at high energies. The asymmetry of photon polarization in the bremsstrahlung due to polarized electrons were investigated in 1992 by Mergl and others^[14] and in 1996 by Haug^[15]. In 2002, Likhachev and others^[16], calculated experimentally the degree of linear and circular polarization of bremsstrahlung photons, and in 2005, Lee and others^[17] investigated the production of bremsstrahlung in the field of heavy atoms at intermediate range of energies. Haug^[18] and Nukel^[19], in 2006, both studied the photon polarization and the spin asymmetry in the bremsstrahlung process in details. In 2009, S.A.Alkhateeb^[20], obtained the relation between the circular polarization of photons and the longitudinal polarization of the scattered electrons in the electromagnetic field of light nuclei.

II-Formulation of the Problem

The bremsstrahlung photons γ are produced in the process of the interaction of electrons with the field of nuclei:

$$e_i^- + N \rightarrow N + e_f^- + \gamma \quad (1)$$

Feynman diagrams of this process are shown in Fig. a.

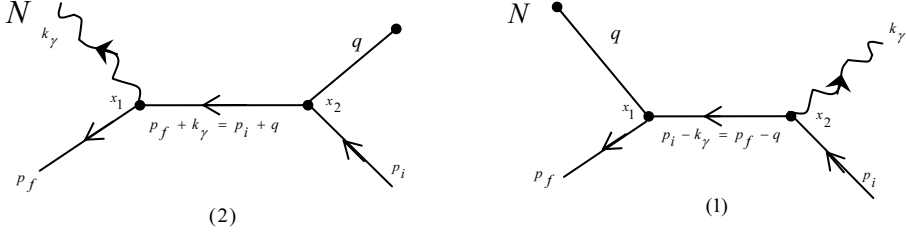


Fig .a

From these diagrams we see that the momentum transfer to the nucleus in the process is $q = p_f + k_\gamma - p_i$. The matrix element (s-matrix) for the process could be written as the sum of two amplitudes corresponding to the above diagrams (1) and (2) and has the form[3]:

$$S_{fi} = e^2 \int \bar{U}_f(x_1) \Gamma(x_1, x_2) U_i(x_2) d^4x_1 d^4x_2 \quad (2)$$

$$\Gamma(x_1, x_2) = A_K(x_1, k_\gamma) G_f(x_1 - x_2) \gamma_4 A_N(x_1) + \gamma_4 A_N(x_2) G_f(x_1 - x_2) A_K(x_2, k_\gamma) \quad (3)$$

Where A_K is the electromagnetic potential of the emitted photon with momentum k_γ , A_N is the potential of the nuclear field, $G_f(x_1 - x_2)$ is the Feynman propagator of the intermediate state between the two points x_2 and x_1 . The potentials A_K and A_N are given in the form[5]:

$$A_K(x, k_\gamma) = \varepsilon^\mu \sqrt{\frac{4\pi}{2\omega V}} (e^{i\vec{k}\cdot\vec{x}} + e^{-i\vec{k}\cdot\vec{x}}) \quad (4)$$

$$A_N(x) = -4\pi Z e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} e^{i\vec{q}\cdot\vec{x}} \quad (5)$$

Where $\varepsilon^\mu = (0, \vec{\varepsilon})$ is the polarization vector of the photon, ω is the photon energy, V is the normalization volume, Z is the atomic number of the target nucleus, the potential $A_N(x)$ given in (4) represent the coulomb field of nucleus. The differential cross-section of the bremsstrahlung with respect to photon energy is given by [4]:

$$\frac{d\sigma}{d\omega} = \frac{Z^2 e^6 m_0^2}{\pi^2} \frac{|p_f|}{|p_i|} \frac{\omega}{|q|^4} F(p_i, p_f, k_\gamma) d\Omega_\varepsilon d\Omega_{k_\gamma} \quad (6)$$

In(6): m_0 is the rest mass of electron, e is the electronic charge. $d\Omega_e$ and $d\Omega_{k_\gamma}$ are the solid angles of the scattered electron and the emitted photon respectively, $F(p_i, p_f, k_\gamma)$ is a function depends on the initial (p_i) and final (p_f) momenta of electron and the photon momentum (k_γ). The final form of this function is given by [20]:

$$F = \frac{-1}{4m_0^2 (p_i k_\gamma)^2 (p_f k_\gamma)^2} \left\{ 4m_0^2 [(p_f k_\gamma) E_f - (p_i k_\gamma) E_i]^2 + [(p_f k_\gamma)^2 + (p_i k_\gamma)^2] \cdot \right. \\ \left. [2(p_i k_\gamma)(p_f k_\gamma) - q^2 m_0^2] + 2(p_i k_\gamma) q^2 (p_f k_\gamma) [E_i^2 + E_f^2 - (p_i p_f)] \right\} \quad (7)$$

here:

$$(p_i k_\gamma) = \omega [E_i - p_i \cos \theta_i] \quad , \quad (p_f k_\gamma) = \omega [E_f - p_f \cos \theta_f] \quad (8)$$

$$q^2 = -[p_i^2 + p_f^2 + \omega^2 + 2\omega p_f \cos \theta_f - 2\omega p_i \cos \theta_i - 2p_i p_f \cos(p_i, p_f)] \quad (9)$$

$$\cos(p_i, p_f) = \cos \theta_i \cos \theta_f + \sin \theta_i \sin \theta_f \cos \phi \quad (10)$$

θ_i, θ_f are the polar angles of p_i, p_f with respect to the photon direction, $\cos(p_i, p_f)$ is the angle between p_i and p_f , and ϕ is the angle between the planes (p_i, k_γ) and (p_f, k_γ) .

III-Analytical Results for the Problem

1- The basic Formula for the Differential Cross-Section

The differential cross-section of bremsstrahlung in the coulomb field of the nucleus can be obtained from the above relations, and the final form of this cross - section is given by:

$$\frac{d\sigma}{d\omega} = \eta \left(\frac{1}{q^4} \right) F_{0e}(\theta_i, \theta_f) d\Omega_e d\Omega_{k_\gamma} \quad (11)$$

Which is known as Bethe - Hietler relation for the bremsstrahlung, where:

$$\eta = \frac{Z^2 \alpha^3}{(2\pi)^2} \left(\frac{1}{\omega} \right) \frac{p_f}{p_i} \quad (12)$$

$$F_{0e}(\theta_i, \theta_f) = \frac{p_i^2 \sin^2 \theta_i}{\Delta_i^2} (4E_f^2 - q^2) + \frac{p_f^2 \sin^2 \theta_f}{\Delta_f^2} (4E_i^2 - q^2) + \frac{2\omega^2}{\Delta_i \Delta_f} \cdot \\ (p_i^2 \sin^2 \theta_i + p_f^2 \sin^2 \theta_f) - \\ \frac{2f(\theta_i, \theta_f)}{\Delta_i \Delta_f} (2E_i^2 - 2E_f^2 - q^2) \quad (13)$$

$$\Delta_i = E_i - p_i \cos \theta_i \quad , \quad \Delta_f = E_f - p_f \cos \theta_f \quad (14)$$

$$f(\theta_i, \theta_f) = p_i p_f \sin \theta_i \sin \theta_f \cos \phi \quad (15)$$

E_i , E_f and ω are the energies of initial and final electron and of the emitted photon, where:

$$\omega = E_i - E_f \quad , \quad E_i = \sqrt{p_i^2 + m_0^2} \quad , \quad E_f = \sqrt{p_f^2 + m_0^2} \quad (16)$$

2- Study of the Electron and Photon Polarization and the Electromagnetic Structure of Target Nucleus

The studying of the interaction between the longitudinally polarized electrons and the electromagnetic field of target nucleus corresponding to the charge distribution (Ze) and the magnetic dipole moment (μ_l) of the nucleus, taking into consideration the circular polarization of the bremsstrahlung photons, gives the following formula for the differential cross-section of the process:

$$d\sigma_{s_\gamma s_e}(\theta_i, \theta_f, Ze, \mu_l) = d\sigma_{s_\gamma s_e}^e(\theta_i, \theta_f, Ze) + d\sigma_{s_\gamma s_e}^m(\theta_i, \theta_f, \mu_l) \quad (17)$$

$$d\sigma_{s_\gamma s_e}^e(\theta_i, \theta_f, Ze) = \frac{1}{2} d\sigma_{0e}(\theta_i, \theta_f) + s_\gamma s_e d\sigma_{1e}(\theta_i, \theta_f) \quad (18)$$

$$d\sigma_{s_\gamma s_e}^m(\theta_i, \theta_f, \mu_l) = \frac{1}{2} d\sigma_{0m}(\theta_i, \theta_f) + s_\gamma s_e d\sigma_{1m}(\theta_i, \theta_f) \quad (19)$$

Where $d\sigma_{0e} = d\sigma_{BH}$ is given by (11) and represent the cross-section due to the charge distribution without studying the polarization of particles and photons.

$$d\sigma_{1e}(\theta_i, \theta_f) = \eta \left(\frac{1}{q^4} \right) F_{1e} d\omega d\Omega_e d\Omega_{k_\gamma} \quad (20)$$

$$d\sigma_{0m}(\theta_i, \theta_f) = 4\eta a(\mu_l) \left(\frac{1}{q^2} \right) F_{0m} d\omega d\Omega_e d\Omega_{k_\gamma} \quad (21)$$

$$d\sigma_{1m}(\theta_i, \theta_f) = 2\eta a(\mu_i) \left(\frac{1}{q^2} \right) F_{1m} d\omega d\Omega_e d\Omega_{k_\gamma} \quad (22)$$

Where I is the spin of target nucleon, $d\sigma_{1e} = d\sigma_{ep}$ is the charged cross-section with studying the polarization, $d\sigma_{0m}$ the magnetic cross-section without studying polarization, and $d\sigma_{1m} = d\sigma_{mp}$ is the magnetic cross-section with studying the polarization. The functions F_{1e} , F_{0m} and F_{1m} are given by [20]:

$$\begin{aligned} F_{1e}(\theta_i, \theta_f) = & \frac{p_i^2 \sin^2 \theta_i}{\Delta_i^2} (E_i p_i \omega \cos \theta_i + E_i p_f \omega \cos \theta_f - E_i \omega^2 + m_0^2 \omega) \\ & + \frac{p_f^2 \sin^2 \theta_f}{\Delta_f^2} (E_i p_i \omega \cos \theta_i + E_i p_f \omega \cos \theta_f + E_f \omega^2 - m_0^2 \omega) \\ & - \frac{2f(\theta_i, \theta_f)}{\Delta_i \Delta_f} (E_i p_i \omega \cos \theta_i + E_i p_f \omega \cos \theta_f) + m_0^2 \omega f(\theta_i, \theta_f) \left(\frac{1}{\Delta_f^2} - \frac{1}{\Delta_i^2} \right) \\ & + \frac{m_0^2 \omega^2}{\Delta_i \Delta_f} (p_f^2 \sin^2 \theta_f - p_i^2 \sin^2 \theta_i) \left(\frac{1}{\omega} + \frac{1}{\Delta_i} - \frac{E_i}{m_0^2} \right) \end{aligned} \quad (23)$$

$$\begin{aligned} F_{0m}(\theta_i, \theta_f) = & (q^2 + 4p_f^2) \frac{p_i^2 \sin^2 \theta_i}{4\Delta_i^2} + (q^2 + 4p_i^2) \frac{p_f^2 \sin^2 \theta_f}{4\Delta_f^2} \\ & - \frac{2f(\theta_i, \theta_f)}{\Delta_i \Delta_f} \left(\frac{q^2}{4} + E_i E_f - m_0^2 \right) + \frac{\omega^2}{2} \left(\frac{q^2}{\Delta_i \Delta_f} + \frac{\Delta_i}{\Delta_f} + \frac{\Delta_f}{\Delta_i} + 2 \right) \end{aligned} \quad (24)$$

$$\begin{aligned} F_{1m}(\theta_i, \theta_f) = & \left(\frac{\omega}{p} \right) \left(\frac{q^2}{4} \left\{ 2E_i \left(\frac{1}{\Delta_i} + \frac{1}{\Delta_f} \right) - m_0^2 \left(\frac{1}{\Delta_i^2} + \frac{1}{\Delta_f^2} \right) + \frac{2}{\Delta_i \Delta_f} (E_i^2 + E_i E_f - 3m_0^2) \right\} \right. \\ & \left. - m_0^2 \left(\frac{p_f^2 + 2\omega \Delta_i}{\Delta_i^2} + \frac{p_i^2 - 2\omega \Delta_f}{\Delta_f^2} - \frac{2(E_i E_f - m_0^2)}{\Delta_i \Delta_f} \right) - \frac{\omega E_i}{2} \left(\frac{\Delta_i}{\Delta_f} - \frac{\Delta_f}{\Delta_i} \right) \right) \end{aligned} \quad (25)$$

The quantity $S_e = \pm 1$ represent the helicity of the initial electron. $S_e = +1$ for the right-longitudinal electron. $S_e = -1$ for the left-longitudinal electron. The quantity $S_\gamma = \pm 1$ represent the circular polarization of bremsstrahlung photon. $S_\gamma = +1$ for the right circular photon polarization. $S_\gamma = -1$ for the left circular photon polarization.

3- The Angular Distribution of the Circular Polarized Bremsstrahlung Photon

The integration of (17), (18) and (19) with respect to the solid angle of the scattered electron $d\Omega_e$ gives the angular distribution of the circular polarized bremsstrahlung photon into the form:

$$d\sigma_{s_\gamma s_e}(\theta_i, Ze, \mu_i) = d\sigma_{s_\gamma s_e}^e(\theta_i, Ze) + d\sigma_{s_\gamma s_e}^m(\theta_i, \mu_i) \quad (26)$$

$$d\sigma_{s_\gamma s_e}^e(\theta_i) = \frac{1}{2} d\sigma_{oe}(\theta_i) + s_\gamma s_e d\sigma_{1e}(\theta_i) \quad (25)$$

$$d\sigma_{s_\gamma s_e}^m(\theta_i) = \frac{1}{2} d\sigma_{om}(\theta_i) + s_\gamma s_e d\sigma_{1m}(\theta_i) \quad (28)$$

Where:

$$d\sigma_{oe}(\theta_i) = 4\pi\eta \Phi_{0e}(\theta_i) d\Omega_{k_\gamma} \quad (29)$$

$$d\sigma_{1e}(\theta_i) = 2\pi\eta \Phi_{1e}(\theta_i) d\Omega_{k_\gamma} \quad (30)$$

$$d\sigma_{om}(\theta_i) = 4\pi\eta a(\mu_i) \Phi_{0m}(\theta_i) d\Omega_{k_\gamma} \quad (31)$$

$$d\sigma_{1m}(\theta_i) = 2\pi\eta a(\mu_i) \Phi_{1m}(\theta_i) d\Omega_{k_\gamma} \quad (32)$$

In the ultra relatives case ($k_\gamma, E_i, E_f \gg m_0 c^2$), the functions $\Phi_{0e}, \Phi_{1e}, \Phi_{0m}, \Phi_{1m}$ are given by:

$$\Phi_{0e}(\theta_i) = \frac{1}{8m_0^2 a^2 \Delta_0} \left\{ 2(1-\gamma) - \frac{7-2\gamma}{\Delta_0} - \frac{1-\gamma^2}{\beta^2 \Delta_0} + \frac{(2-2\gamma+\gamma^2)(2-\Delta_0)}{\Delta_0} \varepsilon_0 \right. \\ \left. + \left(\gamma(2-\gamma) + \frac{2(2-2\gamma+\gamma^2)}{\Delta_0} \right) \frac{\varepsilon}{1-\gamma} - \frac{\gamma}{1-\gamma} \left(3 + \frac{1-\gamma^2}{\beta^2} \right) \frac{\varepsilon_\tau}{\beta} \right\} \quad (33)$$

$$\Phi_{1e}(\theta_i) = \frac{1}{8m_0^2 a^2 \Delta_0} \left\{ 4\gamma + \frac{1-6\gamma}{\Delta_0} - \frac{1-\gamma^2}{\beta^2 \Delta_0} + \frac{\gamma(2-\gamma)(2-\Delta_0)}{(1-\gamma)\Delta_0} L - \frac{\gamma}{1-\gamma} \left(3 + \frac{1-\gamma^2}{\beta^2} \right) \frac{\varepsilon_0}{\beta} \right\} \quad (34)$$

$$\Phi_{0m}(\theta_i) = \frac{-3}{2} + \frac{\gamma^2(1-\gamma-\Delta_0)}{2\beta^2 \Delta_0} - \frac{\gamma^2}{4(1-\gamma)} \varepsilon + \frac{\gamma^2(1-\gamma+\gamma\Delta_0)}{2(1-\gamma)\beta^3} \varepsilon_\tau + \frac{(2-2\gamma+\gamma^2)(2+\Delta_0)}{4(1-\gamma)\Delta_0} \varepsilon_0 \quad (35)$$

$$\Phi_{1m}(\theta_i) = \frac{\gamma}{2} \left\{ -1 + \frac{\gamma(1-\gamma-\Delta_0)}{\beta^2 \Delta_0} - \frac{\gamma}{2(1-\gamma)} \varepsilon + \frac{\gamma(1-\gamma+\gamma\Delta_0)}{(1-\gamma)\beta^3} \varepsilon_\tau + \frac{(2-\gamma)(2+\Delta_0)}{2(1-\gamma)\Delta_0} \varepsilon_0 \right\} \quad (36)$$

where:

$$\gamma = \frac{\varepsilon_i}{E_i}, \quad a = \frac{E_i}{m_0 c^2}, \quad \Delta_0 = 1 - \cos \theta_i \quad (37)$$

$$\beta = \sqrt{(1-\gamma)^2 + 2\gamma\Delta_0}, \quad L = 2 \ln \frac{2a(1-\gamma)}{\gamma} \quad (38)$$

$$\varepsilon_0 = 2 \ln [2a(1-\gamma)], \quad \varepsilon_r = \ln \frac{\beta - \gamma + 1}{\beta + \gamma - 1}, \quad \varepsilon = -2 \ln \gamma \quad (39)$$

The second term in (27) and (28) represents the effect of the spin correlation between the electron and photon spins on the charge and magnetic distributions of the target nucleus respectively.

4- Calculation of the Degree of Circular Polarization of the Bremsstrahlung Photons

The degree of circular polarization in the case of the bremsstrahlung emitted from the collision of electron with nuclei having charge distribution and magnetic dipole moment, is give by:

$$P(\theta_i, Ze, \mu_i) = \frac{\left[d\sigma_{s_\gamma s_e}(\theta_i, Ze, \mu_i) \right]_{s_\gamma=1} - \left[d\sigma_{s_\gamma s_e}(\theta_i, Ze, \mu_i) \right]_{s_\gamma=-1}}{\left[d\sigma_{s_\gamma s_e}(\theta_i, Ze, \mu_i) \right]_{s_\gamma=1} + \left[d\sigma_{s_\gamma s_e}(\theta_i, Ze, \mu_i) \right]_{s_\gamma=-1}} \quad (40)$$

Using the relations (26) - (36) we get the following relation for (P):

$$P(\theta_i, Ze, \mu_i) = s_e p_e(\theta_i) \left[\frac{1 + a(\mu_i) R_1(\theta_i)}{1 + a(\mu_i) R_0(\theta_i)} \right] \quad (41)$$

$$\text{Where: } p_e(\theta_i) = \frac{\Phi_{1e}}{\Phi_{0e}}, \quad R_0(\theta_i) = \frac{\Phi_{0m}}{\Phi_{0e}}, \quad R_1(\theta_i) = \frac{\Phi_{1m}}{\Phi_{1e}}$$

$p_e(\theta_i)$ represents the degree of charged circular polarization of bremsstrahlung produced in the collision of longitudinally polarized electrons ($s_e = 1, -1$) with the field of nuclei having zero magnetic dipole moment. If we take the average on the spin state of electron ($s_e = \pm 1$) and the summation over the photon spin ($s_\gamma = \pm 1$), we have the relation:

$$d\sigma(\theta_i, Ze, \mu_i) = d\sigma_{0e}(\theta_i) + d\sigma_{0m}(\theta_i) = d\sigma_{0e} [1 + a(\mu_i) R_0] \quad (42)$$

Which represents the angular distribution of the bremsstrahlung produced by un-polarized electron in the field of a nucleus with spin

I.The second term in (42) gives the magnetic correction to Bethe-Heitler relation for bremsstrahlung. This correction is more effective for the light nuclei, which has a large magnetic moment at large angles of the photon emission.

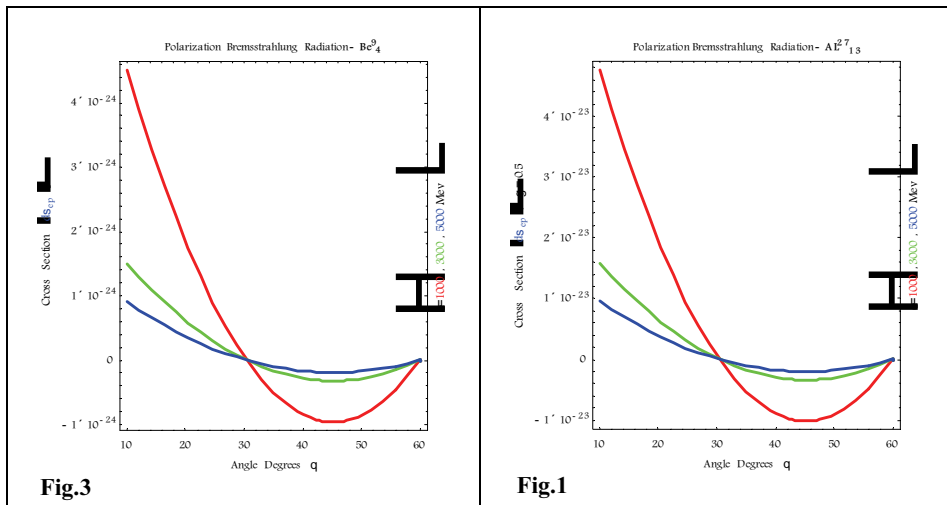
5- Numerical Application to some Nuclei (Results and Discussion)

We shall now consider some applications of the obtained formulas to particular nuclei. The electric and magnetic scattering cross-sections for the two nuclei (Al_{13}^{27}) and (Be_4^9) at different values of energy $\epsilon = (1000,3000,5000 \text{ Mev})$ and angles $\theta = (10^0,30^0,60^0)$, and for the polarized and unpolarized cases are considered, and we have the following results:

1- the polarized electric cross-sections for the two nuclei are decreased with increasing both energies and scattering angles, such that $d\sigma_e$ is more effective at energy $\epsilon = 1000 \text{ Mev}$ and angle $\theta = 10^0$ [Fig.1+3], for ex:

$$d\sigma_{ep} = 6.7982 \times 10^{-31} \text{ (at } \epsilon = 1000 \text{ Mev, } \theta = 10^0 \text{) ,}$$

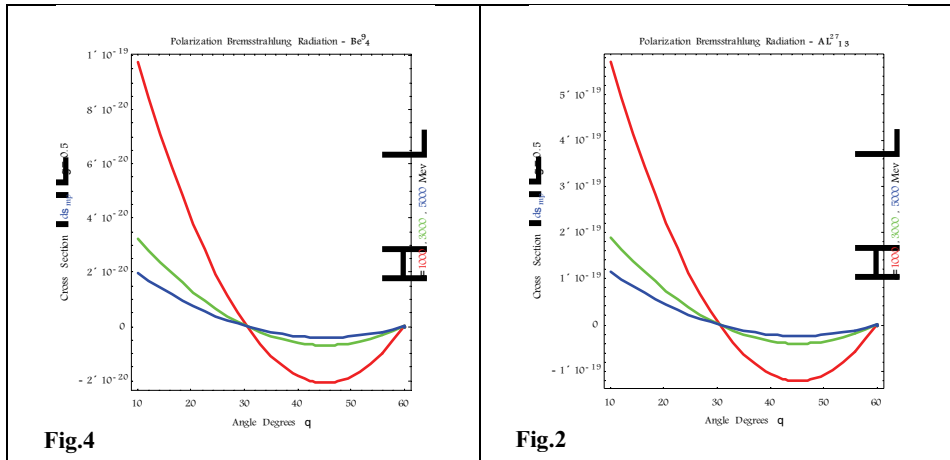
$$d\sigma_{ep} = 4.72937 \times 10^{-36} \text{ (at } \epsilon = 5000 \text{ Mev, } \theta = 60^0 \text{)}$$



2- the polarized magnetic cross-sections for the two nuclei are also decreased with increasing both energies and scattering angles, such that $d\sigma_e$ is more effective at energy $\epsilon = 1000 \text{ Mev}$ and angle $\theta = 10^0$ [Fig(2) + (4)], for ex:

$$d\sigma_{mp} = 5.7122 \times 10^{-19} \text{ (at } \epsilon = 1000 \text{ Mev, } \theta = 10^0 \text{) ,}$$

$$d\sigma_{mp} = 1.05474 \times 10^{-22} \text{ (at } \epsilon = 5000 \text{ Mev, } \theta = 60^0 \text{)}$$

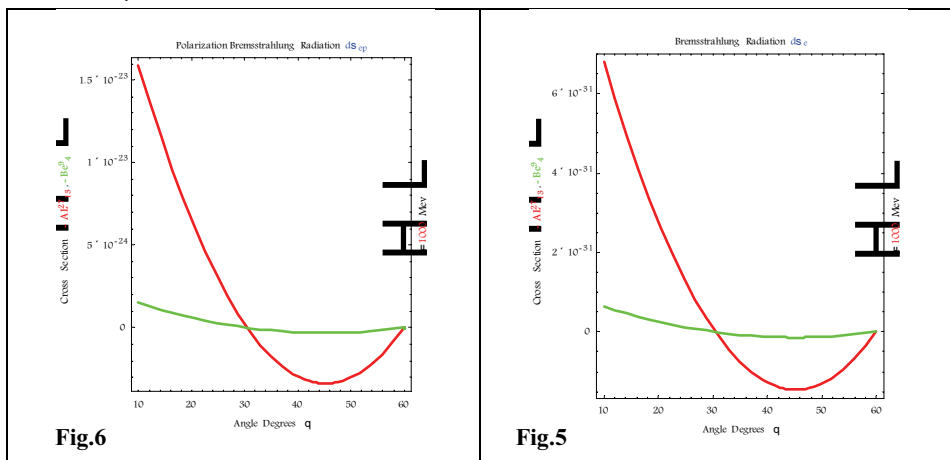


3- comparing the polarized $d\sigma_{ep}$ and unpolarized $d\sigma_e$ electric cross-sections for the two nuclei (Al_{13}^{27}) and (Be_4^9) [Fig(5) + (6)], we see that:

- i) the values of cross-sections for (Al_{13}^{27}) are larger than the value for (Be_4^9).
- ii) the value of polarized $d\sigma_{ep}$ electric cross-section for the two nuclei is larger than the value for the unpolarized $d\sigma_e$ cross-section, for ex:

$$d\sigma_e = 6.7982 \times 10^{-31} \text{ (at } \varepsilon = 1000 \text{ Mev, } \theta = 10^0 \text{) ,}$$

$$d\sigma_{ep} = 4.75822 \times 10^{-23} \text{ (at } \varepsilon = 1000 \text{ Mev, } \theta = 10^0 \text{)}$$



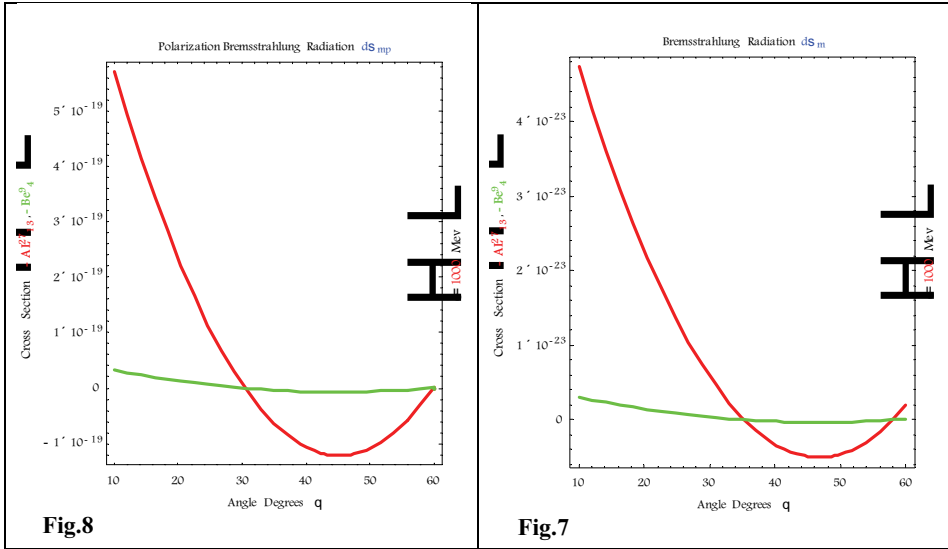
4- comparing the polarized $d\sigma_{mp}$ and un polarized $d\sigma_m$ magnetic cross-sections for the two nuclei (Al_{13}^{27}) and (Be_4^9) [Fig(7) + (8)],we see that:

i) the value of cross-sections for (Al_{13}^{27}) are larger than the value for (Be_4^9).

ii) the value of polarized $d\sigma_{mp}$ magnetic cross-section for the two nuclei is larger than the value for the un polarized $d\sigma_m$ cross-section, for ex:

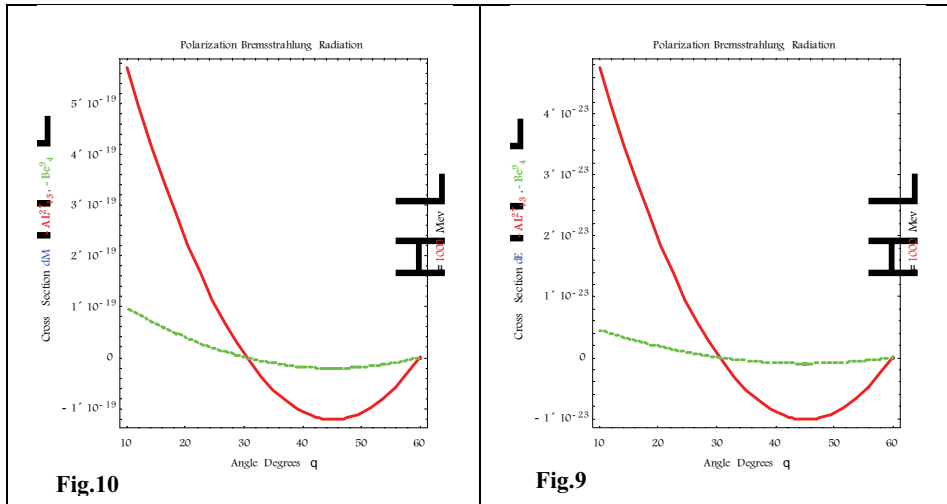
$$d\sigma_m = 4.73899 \times 10^{-23} \text{ (at } \varepsilon = 1000 \text{ Mev, } \theta = 10^0 \text{),}$$

$$d\sigma_{mp} = 5.7122 \times 10^{-19} \text{ (at } \varepsilon = 1000 \text{ Mev, } \theta = 10^0 \text{)}$$



5- comparing the total electric cross-section $dE = d\sigma_e + d\sigma_{ep}$ and the total magnetic cross-section $dM = d\sigma_m + d\sigma_{mp}$ for the two nuclei [Fig(9) + (10)],we see that:

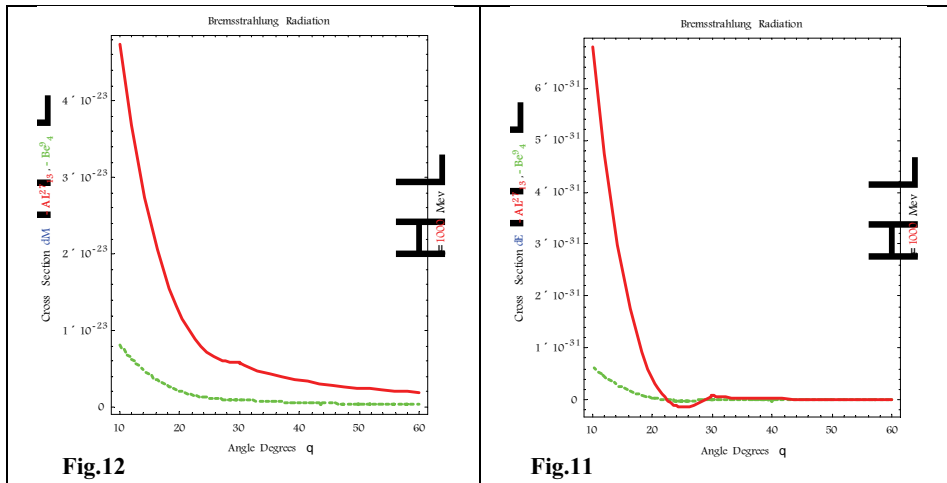
The values of $d\sigma_E$ and $d\sigma_M$ for the (Al_{13}^{27}) nucleus are larger than that for the (Be_4^9) nucleus, and have the largest value at $\varepsilon = 1000 \text{ Mev, } \theta = 10^0$.



6- comparing between the electric and magnetic parts of the cross-section for the two nuclei [Fig (11) + (12)], where $dE = d\sigma_e + d\sigma_{ep}$, $dM = d\sigma_m + d\sigma_{mp}$. We see that:

i) the magnetic part is more effective (i.e has larger values) in the polarized and un polarized cross for the two nuclei.

ii) in the polarized case, the values of cross-sections (electric or magnetic) are larger than the unpolarized case, which means that the study of polarization in the bremsstrahlung process gives us best result, and make the process more evident than that if we neglect the polarization effect, a result in consistent with the experimental results.



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الاعتماد الاستقطابي للمقاطع المستعرضة للأشعة الكابحة في استقطاب إلكترونات بواسطة نويات ذات عزوم كهرومغناطيسية متعددة الأقطاب

سعدة عبدالله الخطيب

قسم الرياضيات، كلية العلوم للبنات، جامعة الملك عبدالعزيز

جدة، المملكة العربية السعودية

المستخلص. يهتم هذا المشروع البحثي بدراسة واحدة من أهم العمليات الإلكترونية الكمية وهي عملية الأشعة الكابحة الناتجة عن التفاعل بين الإلكترون ومجال نواة ذات عزوم كهرومغناطيسية متعددة الأقطاب وذلك في حالتين:

١- مع إهمال الاستقطاب الدائري الفوتوني والاستقطاب الطولي للإلكترون المتفاعل.

٢- مع أخذ الاستقطاب الفوتوني والإلكتروني في الاعتبار.

ويشمل البحث استنتاج العلاقات الأساسية الخاصة بعملية الأشعة الكابحة في الحالتين واستخدام بعض الطرق العددية للحصول على النتائج الخاصة بالعملية وتحليل تلك النتائج.

وبإجراء مقارنة بين الحالتين المذكورتين لبيان مدى تأثير دراسة الاستقطاب الفوتوني والإلكتروني على المقاطع المستعرضة للعملية وبالتالي على إمكانية حدوثها، وسوف نطبق الدراسة على إحدى النويات الخفيفة (Be_4^9) وعلى نواة أثقل منها (Al_{13}^{27})، ومقارنة النتائج في الحالتين.